Chapter 9 – Multifactor Models of Risk and Return

Questions to be answered:
• What are the deficiencies of the CAPM as an explanation of the relationship between risk and expected asset returns?
• What is the arbitrage pricing theory (APT) and what are its similarities and differences relative to the CAPM?
• What are the strengths and weaknesses of the APT as a theory of how risk and expected return are related?
• How can the APT be used in the security valuation process?
• How do you test the APT by examining anomalies found with the CAPM?

Chapter 9 - Multifactor Models of Risk and Return

• What are multifactor models and how are they related to the APT?
• What are the steps necessary in developing a usable multifactor model?
• What are the two primary approaches employed in defining common risk factors?
• What are the main macroeconomic variables used in practice as risk factors?

Chapter 9 - Multifactor Models of Risk and Return

• What are the main security characteristic-oriented variables used in practice as risk factors?
• How can multifactor models be used to identify the investment “bets” that an active portfolio manager is make relative to a benchmark?
• How are multifactor models used to estimate the expected risk premium of a security or portfolio?
Arbitrage Pricing Theory (APT)

- CAPM is criticized because of the difficulties in selecting a proxy for the market portfolio as a benchmark
- An alternative pricing theory with fewer assumptions was developed:
  - Arbitrage Pricing Theory

Arbitrage Pricing Theory - APT

Three major assumptions:
1. Capital markets are perfectly competitive
2. Investors always prefer more wealth to less wealth with certainty
3. The stochastic process generating asset returns can be expressed as a linear function of a set of $K$ factors or indexes

Assumptions of CAPM That Were Not Required by APT

APT does not assume
- A market portfolio that contains all risky assets, and is mean-variance efficient
- Normally distributed security returns
- Quadratic utility function
Arbitrage Pricing Theory (APT)

\[ R_i = E(R_i) + b_{i1} \delta_j + b_{i2} \delta_j + \ldots + b_{ik} \delta_k + \varepsilon_i \]

For \( i = 1 \) to \( N \) where:

- \( R_i \) = return on asset \( i \) during a specified time period
- \( E(R_i) \) = expected return for asset \( i \)
- \( b_{ij} \) = reaction in asset \( i \)'s returns to movements in a common factor
- \( \delta_k \) = a common factor with a zero mean that influences the returns on all assets
- \( \varepsilon_i \) = a unique effect on asset \( i \)'s return that, by assumption, is completely diversifiable in large portfolios and has a mean of zero

Arbitrage Pricing Theory (APT)

\[ R_i = E(R_i) + b_{i1} \delta_j + b_{i2} \delta_j + \ldots + b_{ik} \delta_k + \varepsilon_i \]

where:

- \( R_i \) = actual return on asset \( i \) during a specified time period
- \( E(R_i) \) = expected return for asset \( i \) if all the risk factors have zero changes
- \( b_{ij} \) = reaction in asset \( i \)'s returns to movements in a common factor \( j \)
- \( \delta_k \) = a set of common factors or indexes with a zero mean that influences the returns on all assets
- \( \varepsilon_i \) = a unique effect on asset \( i \)'s return that, by assumption, is completely diversifiable in large portfolios and has a mean of zero
- \( N \) = number of assets

Arbitrage Pricing Theory (APT)

\( \delta_t \) Multiple factors expected to have an impact on all assets:
Arbitrage Pricing Theory (APT)

Multiple factors expected to have an impact on all assets:
– Inflation
– Growth in GDP
– Major political upheavals
Arbitrage Pricing Theory (APT)

Multiple factors expected to have an impact on all assets:
– Inflation
– Growth in GNP
– Major political upheavals
– Changes in interest rates

Contrast this with CAPM’s insistence that only beta is relevant
Arbitrage Pricing Theory (APT)

B_j determine how each asset reacts to this common factor. Each asset may be affected by growth in GDP, but the effects will differ. In application of the theory, the factors are not identified. Similar to the CAPM, the unique effects are independent and will be diversified away in a large portfolio.

Arbitrage Pricing Theory (APT)

- APT assumes that, in equilibrium, the return on a zero-investment, zero-systematic-risk portfolio is zero when the unique effects are diversified away.
- The expected return on any asset \( E(R_i) \) can be expressed as:

\[
E(R_i) = \hat{\lambda}_0 + \lambda_1 b_{i1} + \lambda_2 b_{i2} + \ldots + \lambda_k b_{ik}
\]

where:
- \( \hat{\lambda}_0 \) = the expected return on an asset with zero systematic risk.
- \( \lambda_j \) = the risk premium related to the common \( j \)th factor.
- \( b_{ij} \) = the pricing relationship between the risk premium and asset - that is how responsive asset \( i \) is to \( j \)th common factor.
Using the APT

\( \lambda_0 \) = the rate of return on a zero-systematic-risk asset (zero beta) is 3 percent

\( (\lambda_0 = .04) \)

\( \lambda_1 \) = unanticipated changes in the rate of inflation. The risk premium related to this factor is 2 percent for every 1 percent change in the rate

\( (\lambda_1 = .02) \)

\( \lambda_2 \) = percent growth in real GDP. The average risk premium related to this factor is 3 percent for every 1 percent change in the rate

\( (\lambda_2 = .03) \)

Using the APT

\( b_{x1} \) = the response of asset \( X \) to changes in the inflation factor is 0.50

\( (b_{x1} = .50) \)

\( b_{y1} \) = the response of asset \( Y \) to changes in the inflation factor is 2.00

\( (b_{y1} = 2.00) \)

\( b_{x2} \) = the response of asset \( X \) to changes in the GDP factor is 1.50

\( (b_{x2} = 1.50) \)

\( b_{y2} \) = the response of asset \( Y \) to changes in the GDP factor is 1.75

\( (b_{y2} = 1.75) \)

Using the APT

\[ E(R_i) = \lambda_0 + \lambda_1 b_{i1} + \lambda_2 b_{i2} \]

\[ = .04 + (.02)b_{i1} + (.03)b_{i2} \]

\[ E(R_x) = .04 + (.02)(0.50) + (.03)(1.50) \]

\[ = .095 = 9.5\% \]

\[ E(R_y) = .04 + (.02)(2.00) + (.03)(1.75) \]

\[ = .1325 = 13.25\% \]
Security Valuation with the APT: An Example

- Three stocks (A, B, C) and two common systematic risk factors have the following relationship: \( \lambda_i = 0 \)

\[
\begin{align*}
E(R_A) &= (0.8)\lambda_A + (0.9)\lambda_B + (0.3)\lambda_C \\
E(R_B) &= (0.2)\lambda_A + (1.3)\lambda_B \\
E(R_C) &= (0.8)\lambda_A + (0.5)\lambda_C
\end{align*}
\]

If \( \lambda_A = 4\% \) and \( \lambda_B = 5\% \)

- Stock A is overvalued; Stock B and C are two undervalued securities
- Consider the following investment proportions
  - \( W_A = -1.0 \)
  - \( W_B = 0.5 \)
  - \( W_C = 0.5 \)

These investment weights imply the creation of a portfolio that is short two shares of Stock A for each share of Stock B and one share of Stock C held long.

Security Valuation with the APT: An Example

- Riskless arbitrage
  - Requires no net wealth invested initially
  - Will bear no systematic or unsystematic risk but
  - Still earns a profit
- Condition must be satisfied as follow:
  - \( \sum w_i = 0 \) i.e. no net wealth invested
  - \( \sum a_i = 0 \) For all K factors [i.e. no systematic risk] and \( w \) is small for all I [unsystematic risk is fully diversified]
  - \( \sum x_i > 0 \) i.e. actual portfolio return is positive

\( W_i \) the percentage investment in security i
Security Valuation with the APT: An Example

Net Initial Investment:
- Short 2 shares of A: +70
- Purchase 1 share of B: -35
- Purchase 1 share of C: -35

Net investment: +10

Net Exposure to Risk Factors:

<table>
<thead>
<tr>
<th>Stock</th>
<th>Factor 1</th>
<th>Factor 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>-1.0</td>
<td>-1.0</td>
</tr>
<tr>
<td>B</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>C</td>
<td>0.5</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Net risk exposure: 0 0

Net Profit:
\[
[2(35)-2(37.20)]+[37.80-35]+[38.50-35] = 1.90
\]

Security Valuation with the APT: An Example

\[ P_1 = 37.70/1.077 = 34.54 \]
\[ P_2 = 37.00/1.057 = 35.76 \]
\[ P_C = 38.40/1.107 = 35.10 \]

- The price of stock A will be bid down while the prices of stock B and C will be bid up until arbitrage trading in the current market is no longer profitable.

Roll-Ross Study

The methodology used in the study is as follows:
1. Estimate the expected returns and the factor coefficients from time-series data on individual asset returns
2. Use these estimates to test the basic cross-sectional pricing conclusion implied by the APT

- The authors concluded that the evidence generally supported the APT, but acknowledged that their tests were not conclusive
Extensions of the Roll-Ross Study

- Cho, Elton, and Gruber examined the number of factors in the return-generating process that were priced.
- Dhrymes, Friend, and Gultekin (DFG) reexamined techniques and their limitations and found the number of factors varies with the size of the portfolio.

The APT and Stock Market Anomalies

- Small-firm effect
  - Reinganum - results inconsistent with the APT
  - Chen - supported the APT model over CAPM
- January anomaly
  - Gultekin and Gultekin - APT not better than CAPM
  - Burmeister and McElroy - effect not captured by model, but still rejected CAPM in favor of APT

Shanken’s Challenge to Testability of the APT

- If returns are not explained by a model, it is not considered rejection of a model; however if the factors do explain returns, it is considered support.
- APT has no advantage because the factors need not be observable, so equivalent sets may conform to different factor structures.
- Empirical formulation of the APT may yield different implications regarding the expected returns for a given set of securities.
- Thus, the theory cannot explain differential returns between securities because it cannot identify the relevant factor structure that explains the differential returns.
Alternative Testing Techniques

• Jobson proposes APT testing with a multivariate linear regression model
• Brown and Weinstein propose using a bilinear paradigm
• Others propose new methodologies

Multifactor Models and Risk Estimation

In a multifactor model, the investor chooses the exact number and identity of risk factors

\[ R_{it} = a_i + \left( b_{i1} F_{1t} + b_{i2} F_{2t} + \cdots + b_{ik} F_{kt} \right) + \epsilon_{it} \]

\( F_j \) is the period \( t \) return to the \( j \)th designated risk factor and \( R_{it} \) can be measured as either a nominal or excess return to security \( i \).

Multifactor Models and Risk Estimation

Multifactor Models in Practice
• Macroeconomic-Based Risk Factor Models
Multifactor Models and Risk Estimation

Multifactor Models in Practice

• Macroeconomic-Based Risk Factor Models
• Microeconomic-Based Risk Factor Models

Macroeconomic-Based Risk Factor Models

• Security return are governed by a set of broad economic influences in the following fashion:

\[ R_s = \alpha + \beta_1 R_m + \beta_2 MP + \beta_3 DEI + \beta_4 UI + \beta_5 UPR + \beta_6 UTS + \epsilon \]

\( R_m \) = the return on a value weighted index of NYSE-listed stocks
\( MP \) = the monthly growth rate in US industrial production
\( DEI \) = the change in inflation, measured by the US consumer price index
\( UI \) = the difference between actual and expected levels of inflation
\( UPR \) = the unanticipated change in the bond credit spread
\( UTS \) = the unanticipated term structure shift (long term less short term RFR)
Macroeconomic-Based Risk Factor Models

- Predictive ability of a model based on a different set of macroeconomic factors
  - Confidence risk
  - Time horizon risk
  - Inflation risk
  - Business cycle risk
  - Market timing risk

Microeconomic-Based Risk Factor Models

- Specify the risk in microeconomic terms using certain characteristics of the underlying sample of securities

\[ (R_i - R_F) = a_i + b_{1i}(R_{m} - R_F) + b_{2i}SMB + b_{3i}HML + \epsilon_i \]

- SMB (i.e. small minus big) is the return to a portfolio of small capitalization stocks less the return to a portfolio of large capitalization stocks
- HML (i.e. high minus low) is the return to a portfolio of stocks with high ratios of book-to-market values less the return to a portfolio of low book-to-market value stocks

Extensions of Characteristic-Based Risk Factor Models

- Carhart (1997) extends the Fama-French three factor model by including a fourth common risk factor that accounts for the tendency for firms with positive past return to produce positive future return
  - Momentum factor

\[ (R_i - R_F) = a_i + b_{1i}(R_{m} - R_F) + b_{2i}SMB + b_{3i}HML + b_{4i}PR10R + \epsilon_i \]
Extensions of Characteristic-Based Risk Factor Models

- The second type of security characteristic-based method for defining systematic risk exposures involves the use of index portfolio (e.g. S&P 500, Wilshire 5000) as common factors

Extensions of Characteristic-Based Risk Factor Models

- The third one is BARRA Characteristic-based risk factors
  - Volatility (VOL)
  - Momentum (MOM)
  - Size (SZ)
  - Size Nonlinearity (SNL)
  - Trading Activity (TRA)
  - Growth (GRO)
  - Earnings Yield (EYL)
  - Value (VAL)
  - Earnings Variability (EVR)
  - Leverage (LEV)
  - Currency Sensitivity (CUR)
  - Dividend Yield (YLD)
  - Nonestimation Indicator (NEU)

Estimating Risk in a Multifactor Setting

- Estimating Expected Returns for Individual Stocks
  - A Specific set of K common risk factors must be identified
  - The risk premia for the factors must be estimated
  - Sensitivities of the ith stock to each of those K factors must be estimated
  - The expected returns can be calculated by combining the results of the previous steps in the appropriate way
Summary

- APT model has fewer assumptions than the CAPM and does not specifically require the designation of a market portfolio.
- The APT posits that expected security returns are related in a linear fashion to multiple common risk factors.
- Unfortunately, the theory does not offer guidance as to how many factors exist or what their identities might be.

Summary

- APT is difficult to put into practice in a theoretically rigorous fashion. Multifactor models of risk and return attempt to bridge the gap between the practice and theory by specifying a set of variables.
- Macroeconomic variable has been successfully applied.
- An equally successful second approach to identifying the risk exposures in a multifactor model has focused on the characteristics of securities themselves. (Microeconomic approach)

The Internet

Investments Online

http://www.kellogg.northwestern.edu/faculty/korajczy/htm/aptlist.htm
http://www.mba.tuck.dartmouth.edu/pages/faculty/ken.french
Future topics
Chapter 10
• Analysis of Financial Statements